SIMPLICIALLY DRIVEN SIMPLE CONTAGION

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SPREADING PROCESSES CAN AFFECT EACH OTHER

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- -: HIV increases susceptibility to other diseases
- -: unsafe behaviours boost pathogen spread

INTERACTING CONTAGION MODELS

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- simple contagions
- contagion symmetricly coupled

$$A \leftrightarrows B$$

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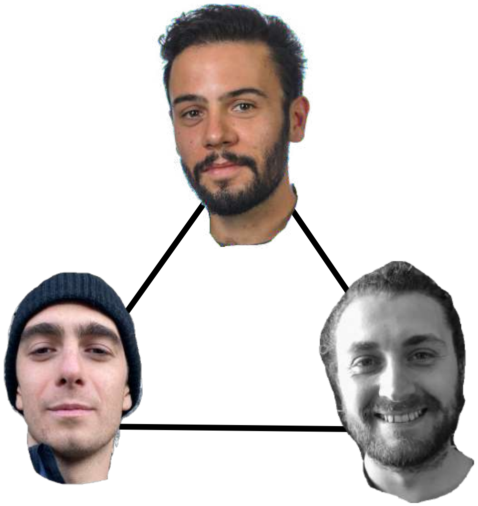
- simple contagions
- contagion symmetricly coupled

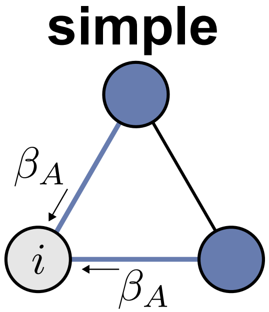
$$A \leftrightarrows B$$

But, behaviours best described by complex contagions, and interaction often not symmetric. We do:

$$A \rightarrow B$$

OUR SPREADING MODEL





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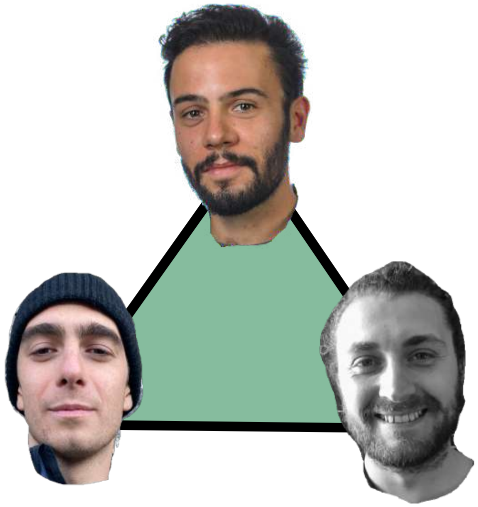

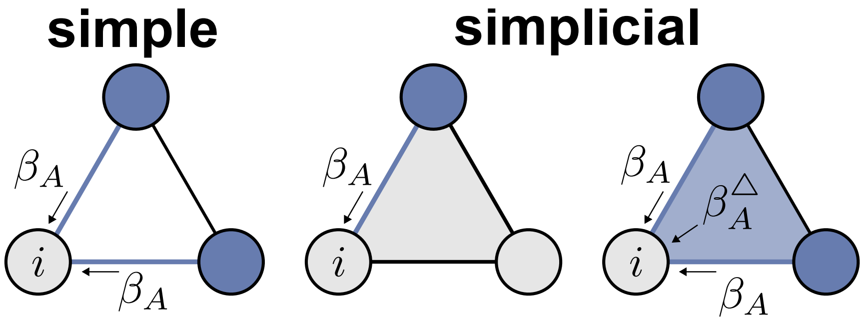

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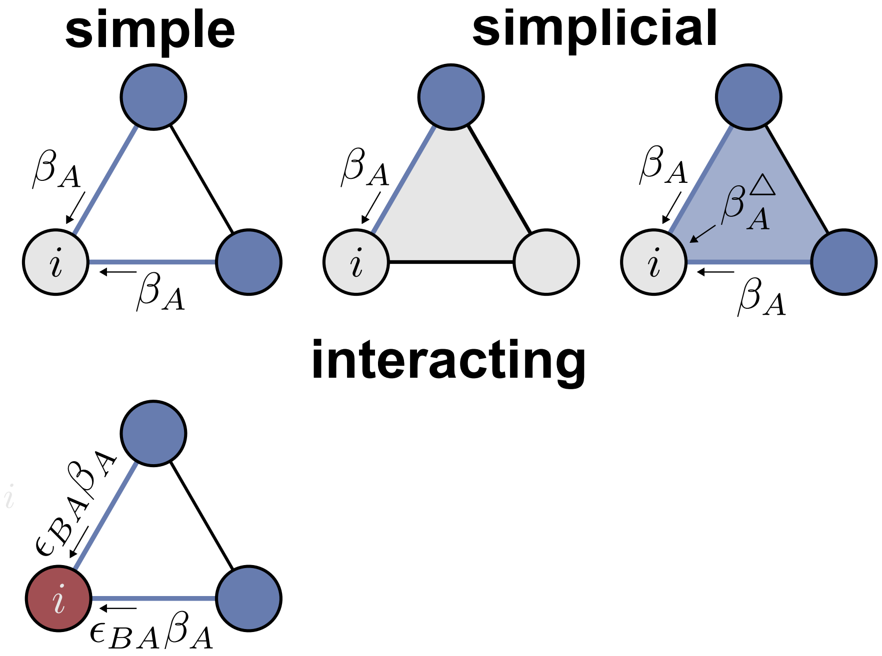

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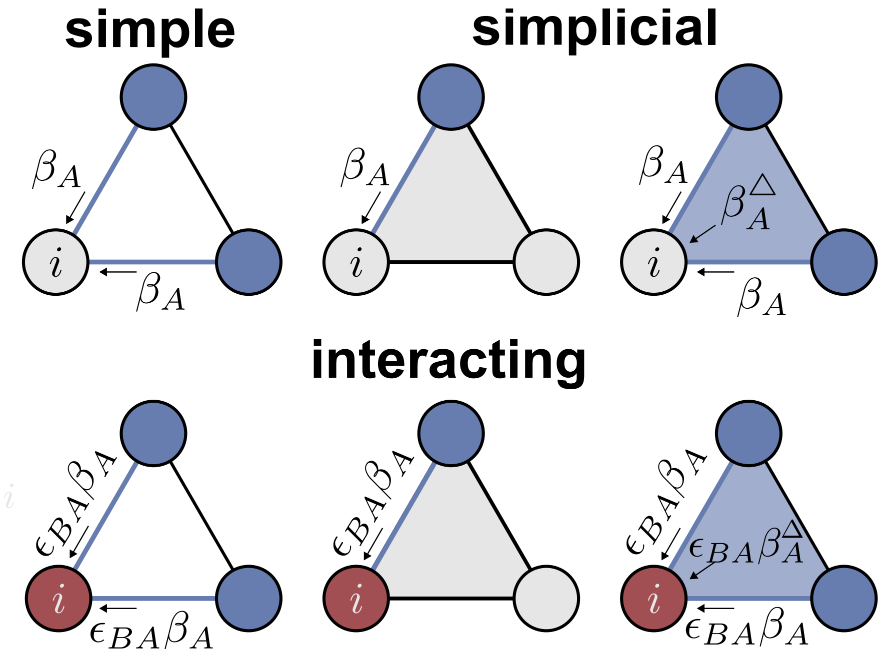

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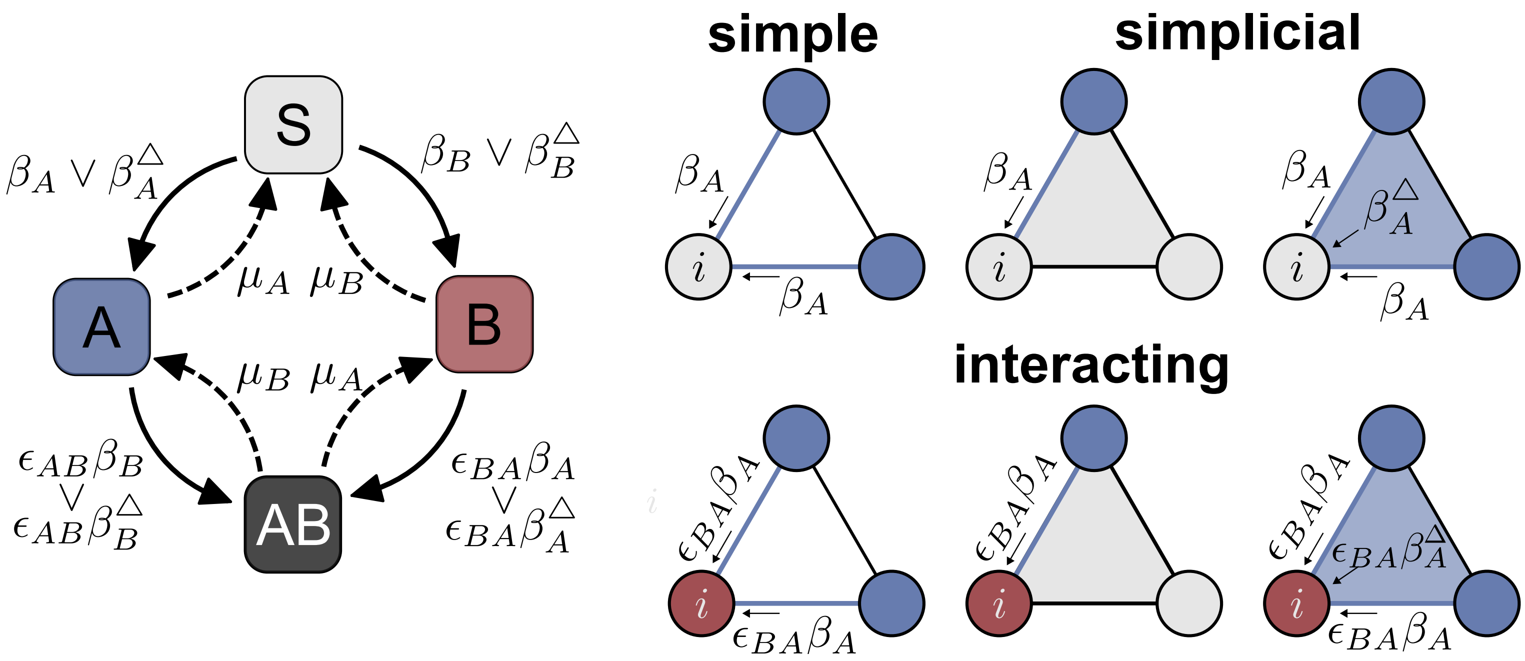


A (UNSAFE BEHAVIOUR)

DRIVES COOPERATIVELY

B (DISEASE)

MEAN-FIELD DESCRIPTION

$$\dot{\rho}_{A_{\text{tot}}} = \rho_{A_{\text{tot}}} \left[-1 + \lambda_A (1 - \rho_{A_{\text{tot}}}) + \lambda_A^{\triangle} \rho_{A_{\text{tot}}} (1 - \rho_{A_{\text{tot}}}) \right]$$

$$\dot{\rho}_{B_{\text{tot}}} = \rho_{B_{\text{tot}}} \left[-1 + \lambda_B (1 - \rho_{B_{\text{tot}}}) + \lambda_B (\epsilon_{AB} - 1)(\rho_{A_{\text{tot}}} - \rho_{AB}) \right]$$

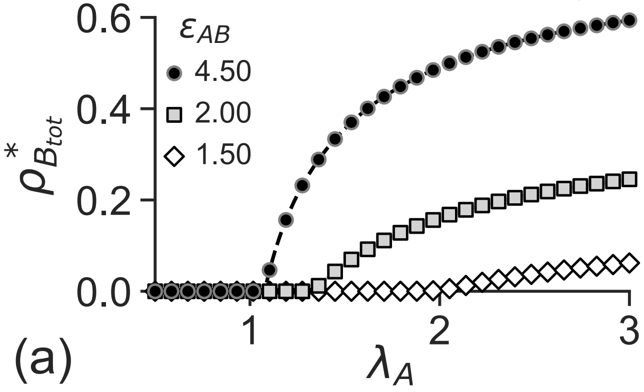
$$\dot{\rho}_{AB} = -2\rho_{AB} + \epsilon_{AB} \lambda_B (\rho_{A_{\text{tot}}} - \rho_{AB}) \rho_{B_{\text{tot}}} + \lambda_A (\rho_{B_{\text{tot}}} - \rho_{AB}) \rho_{A_{\text{tot}}} + \lambda_A^{\triangle} (\rho_{B_{\text{tot}}} - \rho_{AB}) \rho^2_{A}$$

IMPLICIT SOLUTION FOR DRIVEN B

$$\rho^{*,\pm}_{B_{\mathrm{tot}}} = 1 - \frac{1}{\lambda_B} + (\rho^{*,\pm}_{A_{\mathrm{tot}}} - \rho^{*,\pm}_{AB})(\epsilon_{AB} - 1).$$

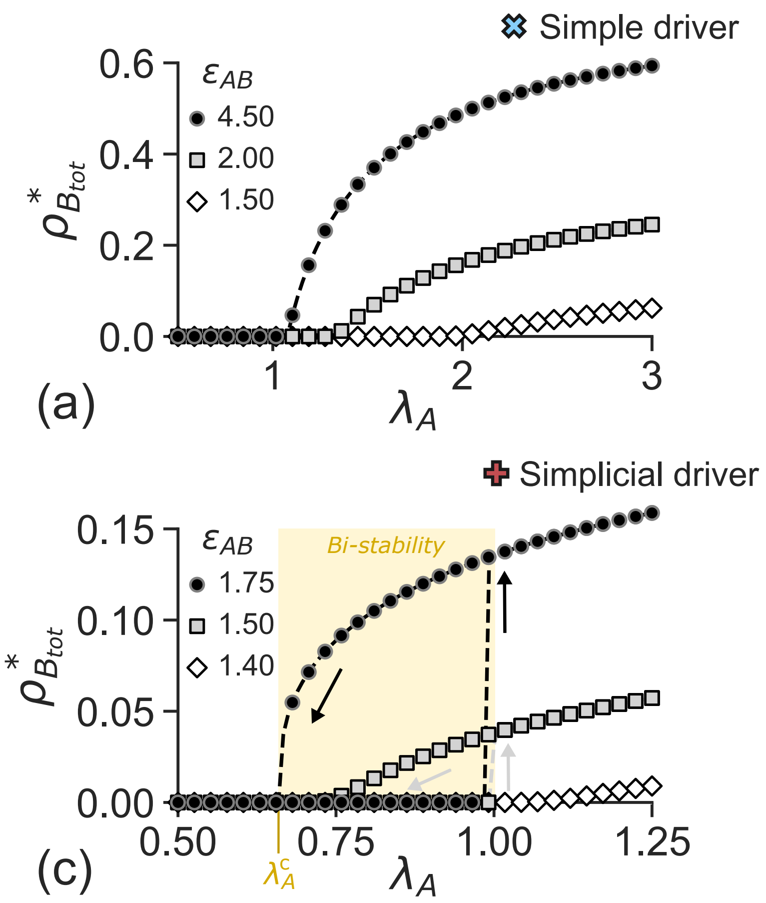
SIMPLE CONTAGION B

Simple driver

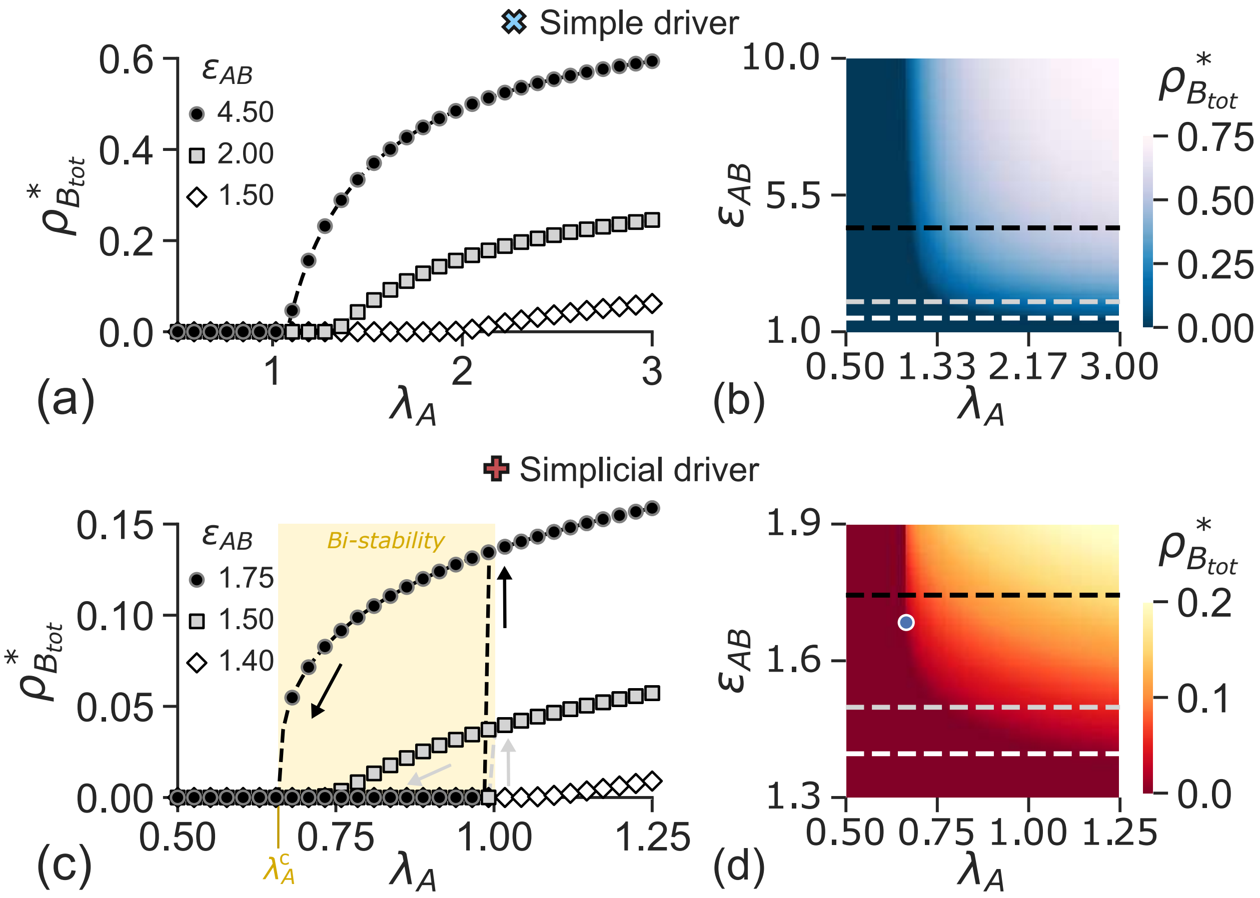


Simplicial driver

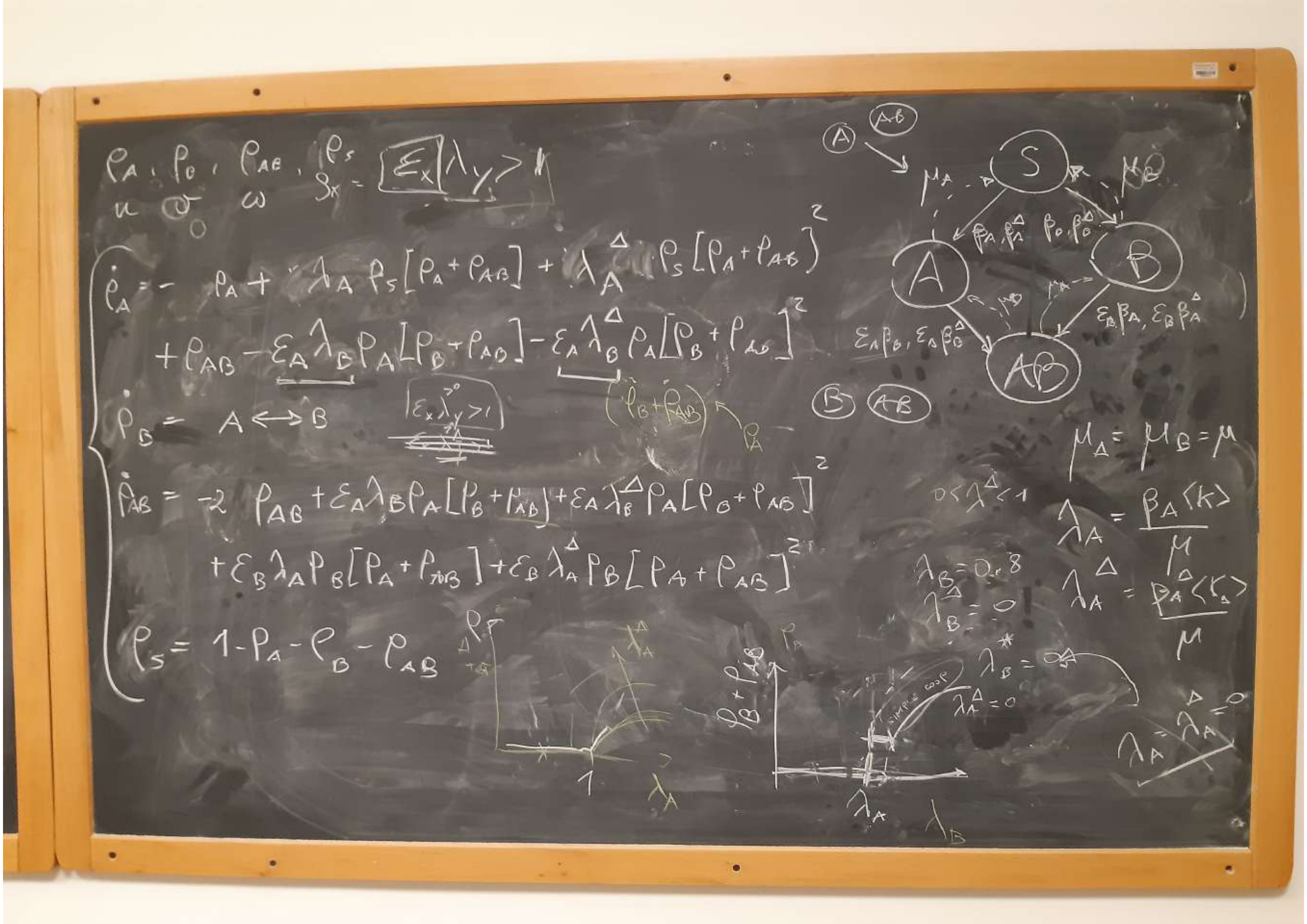
SIMPLE CONTAGION B GOES BOOM..!



SIMPLE CONTAGION B GOES



CRITICAL VALUE OF THE DRIVING E_{AB}?



ρA, ρB, ρAB, ρs
εx λy > 1
ρ̇A = − ρA + λA ρs [ρA + ρAB] + λA^Δ ρs [ρA + ρAB]^2
+ ρAB − εA λB ρA [ρB + ρAB] − εA λB^Δ ρA [ρB + ρAB]^2
ρ̇B = A ⟷ B
ρ̇AB = −2 ρAB + εA λB ρA [ρB + ρAB] + εA λB^Δ ρA [ρB + ρAB]^2
+ εB λA ρB [ρA + ρAB] + εB λA^Δ ρB [ρA + ρAB]^2
ρs = 1 − ρA − ρB − ρAB
S
A
B
AB
μA = μB = μ
0 < λ^Δ < 1
λA = βA ⟨k⟩ / μ
λA^Δ = βA^Δ ⟨kΔ⟩ / μ
λB = 0.8
λB^Δ = 0

$$\dot\rho_A = -\rho_A + \lambda_A \rho_S (\rho_A + \rho_{AB}) + \rho_{AB} - \varepsilon_{A\to B}\lambda_B \rho_A (\rho_B + \rho_{AB})$$

$$\dot\rho_B^{A\leftrightarrow B} =$$

$$\dot\rho_{AB} = -2\rho_{AB} + \varepsilon_{A\to B}\lambda_B\rho_A(\rho_B+\rho_{AB}) + \varepsilon_{B\to A}\lambda_A\rho_B(\rho_A+\rho_{AB})$$

$$\to \dot\rho_B + \dot\rho_{AB} = -\rho_B + \lambda_B(1-\rho_A-\rho_B-\rho_{AB})(\rho_B+\rho_{AB}) + \varepsilon_{A\to B}\lambda_B\rho_A(\rho_B+\rho_{AB}) - \rho_{AB}$$

$$\rho_A = \rho_A^* > 0$$

$$\dot\rho_{B_{TOT}} = \rho_{B_{TOT}}\left[\lambda_B(1-\rho_A^*-\rho_{B_{TOT}}) + \varepsilon_{A\to B}\lambda_B\rho_A^* - 1\right] \stackrel{!}{=} 0$$

$$\lambda_B|_{\varepsilon_{A\to B}>1} > \lambda_B|_{\varepsilon=1} \qquad \left[(1-c) + \varepsilon c - 1\right] = \rho$$

$$\lambda_{B|A_c} > \lambda_{B_c}$$

$$\rho_{B_{TOT}}^* = 1 - \frac{1}{\lambda_B} + \rho_A^*(\varepsilon_{A\to B}-1) \quad \nearrow \; + k$$

$$\lambda_B^\Delta = 0 = \lambda_A^\Delta$$

$$\rho_S = 1-\rho_A-\rho_B-\rho_{AB}$$

$$\rho_B+\rho_{AB} =$$

$$\rho_A, \rho_B, \rho_{AB}$$

$$\rho_A + \rho_{AB} = 0$$

$$\rho_A = -\rho_{AB}$$

$$\nearrow \; \rho_{B_{TOT}}^* = \rho$$

$$\varepsilon_{A\to B} = 1 \Rightarrow SIS_B : \rho_B^* > 0$$

$$\varepsilon > 1: \quad [1 + \rho_A^*(\varepsilon-1)] = 1^+$$

$$1^+ - \frac{1}{\lambda} > 0$$

$$\rho_B = \frac{\lambda_B \rho_{BTOT}(1-\rho_{ATOT}) + \rho_{BTOT}}{2 + \lambda_B\rho_{BTOT} + \varepsilon_{B\to A}\lambda_A\rho_{ATOT} - \varepsilon_{B\to A}\lambda_A^{\Delta}\rho_{ATOT}^2}$$

$$\dot\rho_{AB} = -2\rho_{AB} + \varepsilon_A\lambda_B\rho_A\rho_{BTOT} + \varepsilon_B\lambda_A\rho_B\rho_{ATOT} + \varepsilon_B\lambda_A^{\Delta}\rho_B\rho_{ATOT}^2$$

$$\dot\rho_A = -\rho_A + \lambda_A(1-\rho_A-\rho_{BTOT})\rho_{ATOT} + \lambda_A^{\Delta}(1-\rho_A-\rho_{BTOT})\rho_{ATOT}^2 + (\rho_{ATOT}-\rho_A) - \varepsilon_A\lambda_B\rho_A\rho_{BTOT}$$

$$0 = \left[\rho_{ATOT}\lambda_A(1-\rho_{BTOT}) + \rho_{ATOT}^2\lambda_A^{\Delta}(1-\rho_{BTOT})\right] + \rho_{ATOT} + \rho_A\left[-\lambda_A\rho_{ATOT} - \lambda_A^{\Delta}\rho_{ATOT}^2 - 2 - \varepsilon_A\lambda_B\rho_{BTOT}\right]$$

$$\rho_A^{*} = \frac{\rho_{ATOT}\lambda_A(1-\rho_{BTOT}) + \rho_{ATOT}^2\lambda_A^{\Delta}(1-\rho_{BTOT})}{\lambda_A\rho_{ATOT} + \lambda_A^{\Delta}\rho_{ATOT}^2 + 2 + \varepsilon_A\lambda_B\rho_{BTOT}}$$

$\bar\varepsilon_A = \varepsilon_{AB} - 1$

IF COOP $\bar\varepsilon_A > 0$

$$\frac{2 + \varepsilon_{AB}\lambda_B\Lambda_B^- + \varepsilon_{AB}2\lambda_B\bar\varepsilon_A\left(1-\frac{1}{\sqrt{\lambda_A^{\Delta}}}\right) + \varepsilon_{AB}\left(\sqrt{\lambda_A^{\Delta}}-1\right)}{2\varepsilon_{AB}\lambda_B\bar\varepsilon_A} > 0$$

$2\varepsilon_{AB}\lambda_B\bar\varepsilon_A > 0$ IF COOP

$$\underbrace{2}_{C'} + \varepsilon_{AB}\underbrace{\left[\lambda_B\Lambda_B^- + \sqrt{\lambda_A^{\Delta}} - 1 - 2\lambda_B\left(1-\frac{1}{\sqrt{\lambda_A^{\Delta}}}\right)\right]}_{B'} + \varepsilon_{AB}^2\underbrace{2\lambda_B\left(1-\frac{1}{\sqrt{\lambda_A^{\Delta}}}\right)}_{A'} > 0$$

$$\varepsilon_{AB}^{*} = \frac{-B' \pm \sqrt{B'^2 - 4AC}}{2A}$$

$$B^2 - 4AC = \left[\lambda_B\Lambda_B^- + \sqrt{\lambda_A^{\Delta}} - 1\right]^2 + 4\lambda_B^2\left(1-\frac{1}{\sqrt{\lambda_A^{\Delta}}}\right)^2 - 2\left[\lambda_B\Lambda_B^- + \sqrt{\lambda_A^{\Delta}}-1\right]\left[2\lambda_B\left(1-\frac{1}{\sqrt{\lambda_A^{\Delta}}}\right)\right] - 16\lambda_B^2\left(1-\frac{1}{\sqrt{\lambda_A^{\Delta}}}\right)^2$$

$$[\ ]^2 - 12(\ )^2 - 2[\ ][\ ]$$

$$\lambda_B\Lambda_B^- = (\lambda_B - 1)$$

$\lambda_\Delta > 1$

$$\Lambda_B^- = 1 - \frac{1}{\lambda_B}$$

$(\varepsilon_{A\to B} - 1)$

$$0 \doteq \rho_{A_{tot}} \lambda_A \left[\rho_B(\varepsilon_{B\to A}-1) - \rho_{A_{tot}} + \Lambda_A\right] + \rho_{A_{tot}}^2 \lambda_A^\Delta \left[1-\rho_{A_{tot}} + (\varepsilon_{B\to A}-1)\rho_B\right]$$

$/\rho_{A_{tot}}$

$$0 = \lambda_A\left[\rho_B E_B^- - \rho_{A_{tot}} + \Lambda_A^-\right] + \rho_{A_{tot}}\lambda_A^\Delta\left[1-\rho_{A_{tot}} + E_B^-\rho_B\right]$$

$$= \underbrace{\lambda_A\left[\rho_B E_B^- + \Lambda_A^-\right]}_{C} + \rho_{A_{tot}}\underbrace{\left[-\lambda_A + \lambda_A^\Delta(1+E_B^-\rho_B)\right]}_{B} - \underbrace{\rho_{A_{tot}}^2\lambda_A^\Delta}_{A=-\lambda_A^\Delta}$$

$$B^2-4AC = \lambda_A^2 + \lambda_A^{\Delta 2}(1+E_B^-\rho_B)^2 - 2\lambda_A\lambda_A^\Delta(1+E_B^-\rho_B) + 4\lambda_A^\Delta\lambda_A(\rho_B E_B^- + \Lambda_A^-)$$

$$+ 2\lambda_A\lambda_A^\Delta(1+E_B^-\rho_B) + \underbrace{4\lambda_A\lambda_A^\Delta\left(-\frac{1}{\lambda_A}\right)}_{-4\lambda_A^\Delta}$$

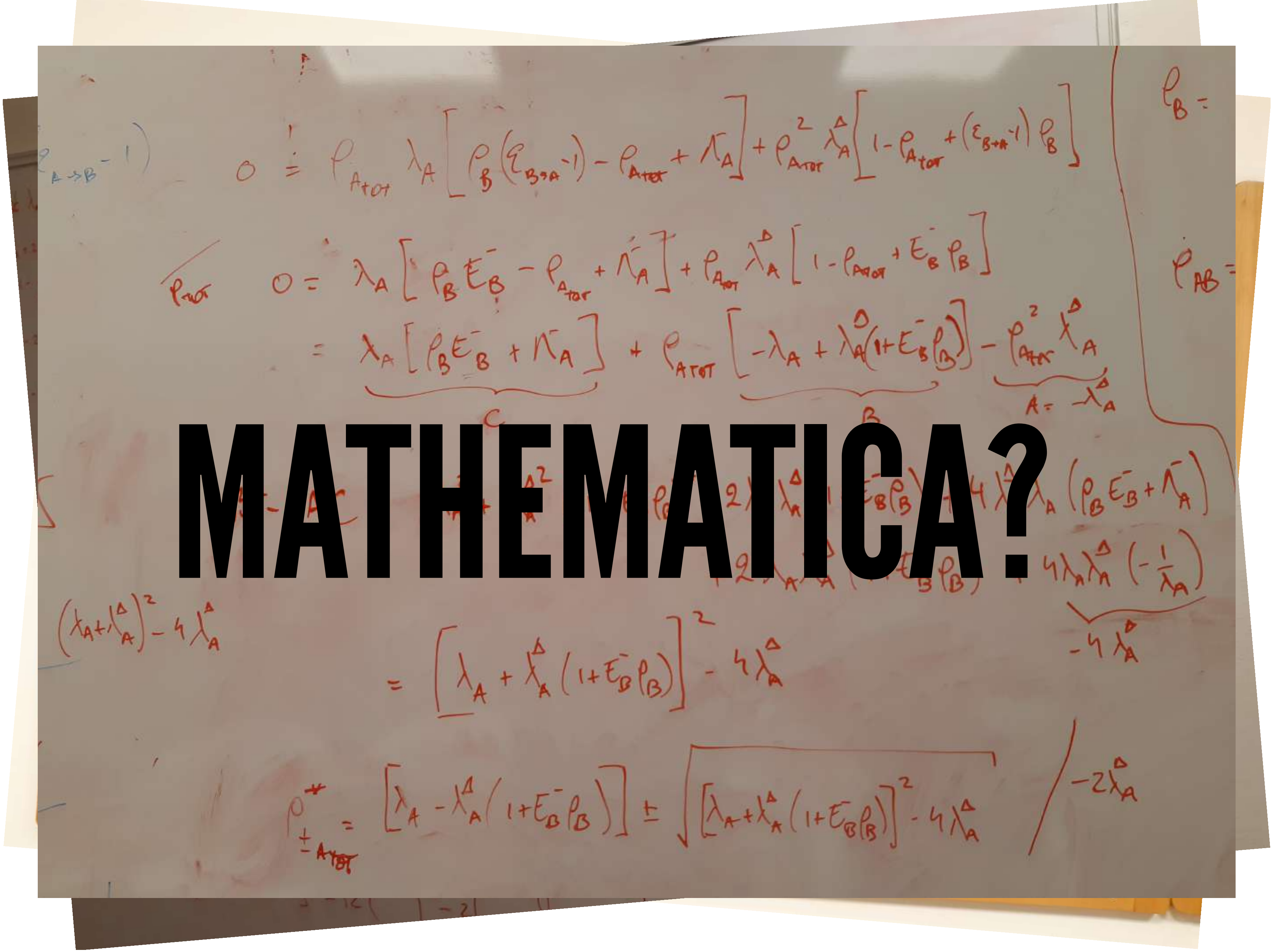
$(\lambda_A+\lambda_A^\Delta)^2 - 4\lambda_A^\Delta$

$$= \left[\lambda_A + \lambda_A^\Delta(1+E_B^-\rho_B)\right]^2 - 4\lambda_A^\Delta$$

$$\rho^*_{\pm A_{tot}} = \frac{\left[\lambda_A - \lambda_A^\Delta(1+E_B^-\rho_B)\right] \pm \sqrt{\left[\lambda_A+\lambda_A^\Delta(1+E_B^-\rho_B)\right]^2 - 4\lambda_A^\Delta}}{-2\lambda_A^\Delta}$$

$\dot\rho_B =$

$\dot\rho_{AB} =$

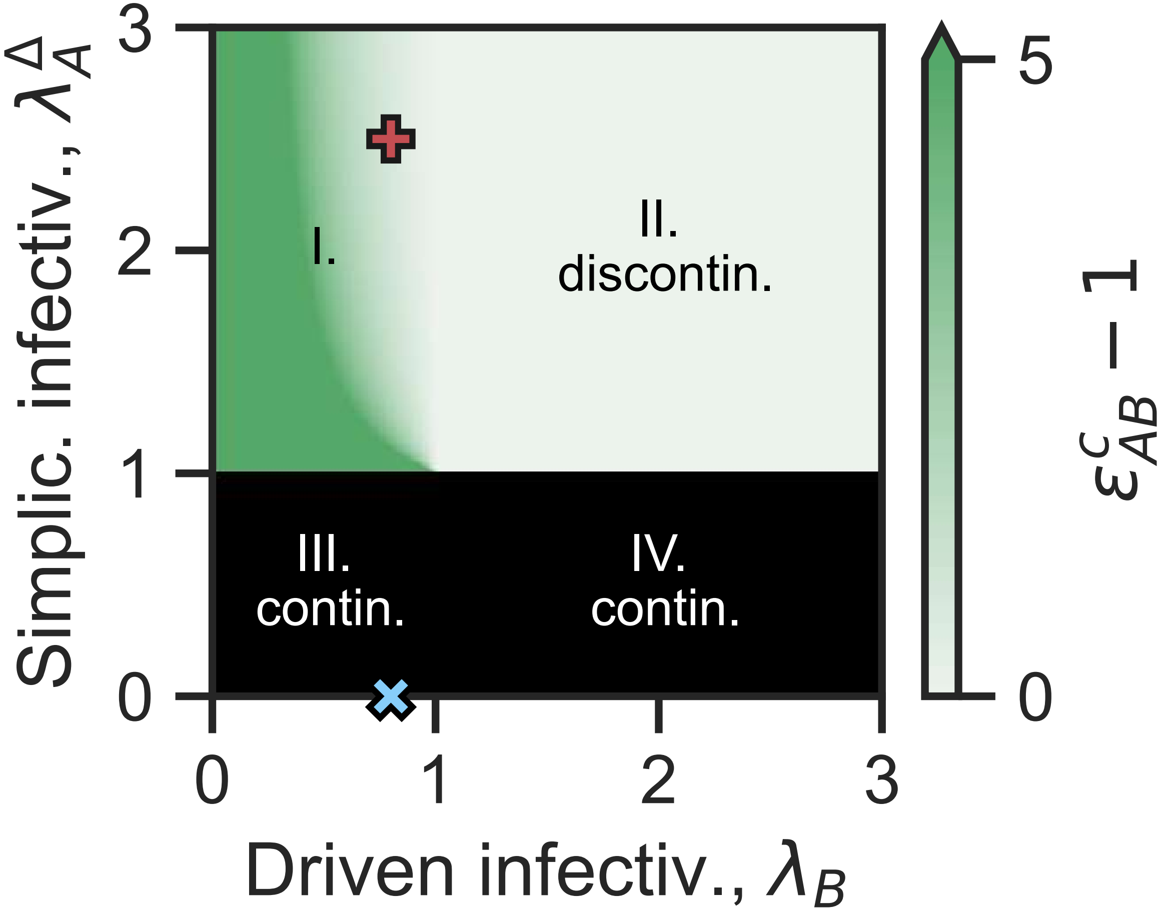

MATHEMATICA?

CRITICAL DRIVING STRENGTH

$$\epsilon^c_{AB} = \begin{cases} \dfrac{\sqrt{\lambda_A^{\triangle}} - \lambda_B}{\left(\sqrt{\lambda_A^{\triangle}} - 1\right)\lambda_B} & \text{in region I} \\ 1 & \text{in region II} \end{cases}$$

above ϵ^c_{AB}: 

PHASE DIAGRAM



EFFECTIVE FORMALISM

simple contagion

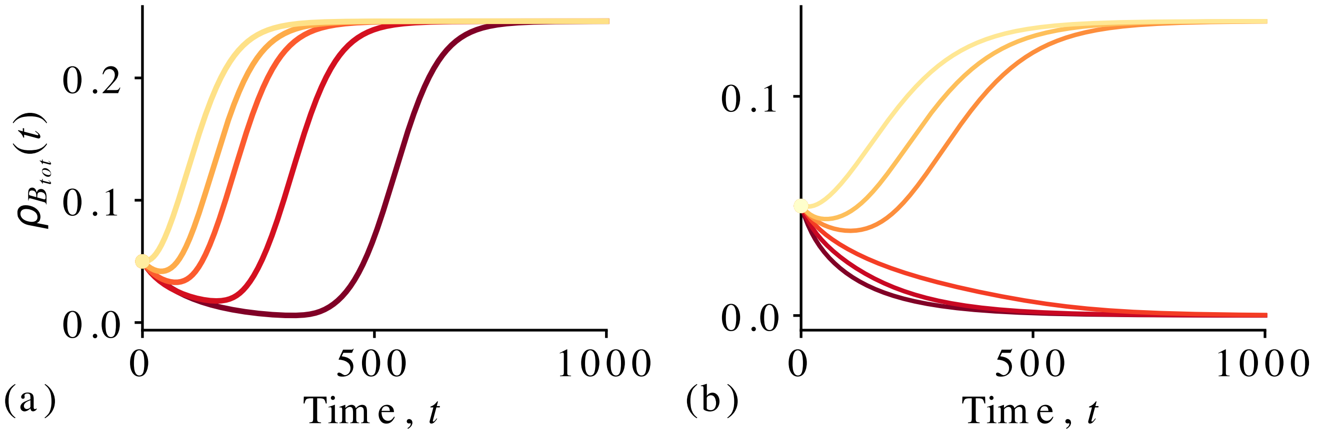
$$\dot{\rho}_{B_{\text{tot}}} = -\rho_{B_{\text{tot}}} + \tilde{\lambda}_B \, \rho_{B_{\text{tot}}} [1 - \rho_{B_{\text{tot}}}],$$

with effective infectivity

$$\tilde{\lambda}_B = \lambda_B + \lambda_B (\epsilon_{AB} - 1) \frac{1}{1 - \rho_{B_{\text{tot}}}} \rho_A .$$

OBSERVING ONLY THE DRIVEN B

bistability in B by changing initial condition of A



TAKE HOME

- Simple drives simple: continuous
- Simplicial drives simple: DIScontinuous
- Effective formalism
- Observing only driven contagion..

QUESTIONS?

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